Quiz
Precalc Day 80/81

Sec. 7.5 Right Triangles and Inverse Trigonometric Functions

We define:

- the arc sine or inverse sine function as $\arcsin x = \sin^{-1} x$ = The angle in a right triangle whose sine is x
- the arc cosine or inverse cosine function as $\arccos x = \cos^{-1} x$ = The angle in a right triangle whose cosine is x
- the arc tangent or inverse tangent function as $\arctan x = \tan^{-1} x$ = The angle in a right triangle whose tangent is x.

This means that for an angle θ in a right triangle (other than the right angle),

$$\sin\theta = x \text{ means } \theta = \sin^{-1} x$$
$$\cos\theta = x \text{ means } \theta = \cos^{-1} x$$
$$\tan\theta = x \text{ means } \theta = \tan^{-1} x.$$

Ex: Find θ, an angle in a right triangle if:

a. $\sin\theta = .7324$ b. $\tan\theta = -1.2368$ c. $\cos\theta = 1.2347$ d. $\cos\theta = \frac{\sqrt{3}}{2}$

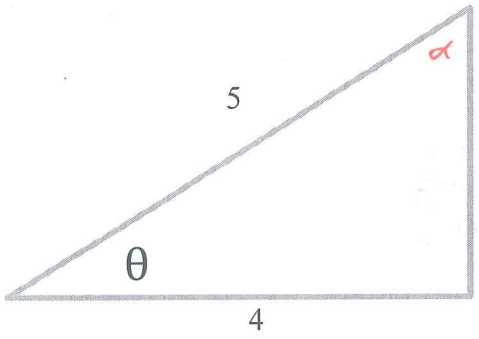
$\theta = \sin^{-1}(.7324)$ or $\theta = \arcsin(.7324)$ $\theta = 47.088^\circ$

$\theta = \tan^{-1}(-1.2368)$ or $\theta = \arctan(-1.2368)$ $\theta = -51.043^\circ$ (not in RT Δ)

$\theta = \cos^{-1}(1.2347)$ Does not exist $-1 \le \cos\theta \le 1$

(without calculator) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (What angle has an x-value of $\frac{\sqrt{3}}{2}$?) $\theta = 30^\circ$

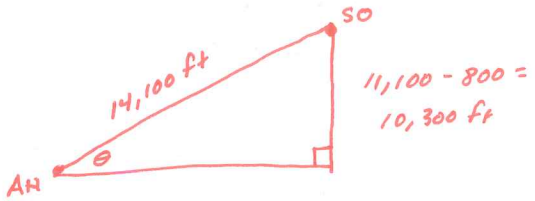
Ex: Use the inverse sine function to find the angles in the figure.



$\sin\theta = \frac{o}{h}$ $\sin\theta = \frac{3}{5}$ $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ $\theta = 36.870^\circ$

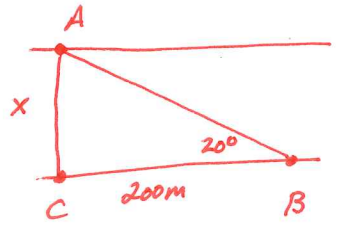
$\sin\alpha = \frac{o}{h}$ $\sin\alpha = \frac{4}{5}$ $\alpha = \sin^{-1}\left(\frac{4}{5}\right)$ $\alpha = 53.130^\circ$

Ex: A straight trail leads from the Alpine Hotel, elevation 800 feet, to a scenic overlook, elevation 11,100 feet. The length of the trail is 14,100 feet. What is the inclination of the trail?



$\sin\theta = \frac{10,300}{14,100}$ $\theta = \sin^{-1}\left(\frac{10,300}{14,100}\right)$ $\theta = 46.928^\circ$

Ex: A surveyor can measure the width of a river by setting up a transit at a point C on one side of the river and taking a sighting of a point A on the other side. After turning through an angle of 90 degrees at C, the surveyor walks a distance of 200 meters to point B. Using the transit at B, the angle B is measured and found to be 20 degrees. What is the width of the river rounded to the nearest meter?



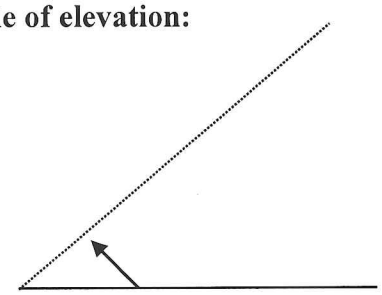
$\tan 20^\circ = \frac{x}{200m}$

$200 \tan 20^\circ = x$

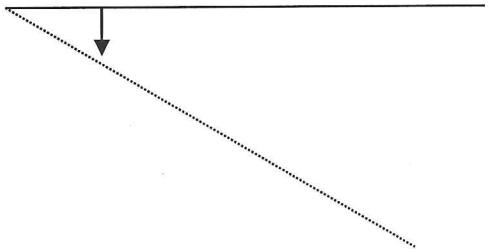
$72.949m = x$

$73m = x$

Angle of elevation:

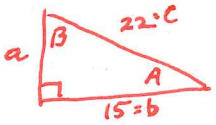


Angle of Depression:



Ex: Find the missing sides and angles in the right triangle, where a is the side across angle A, b across from B, and c across from C.

a. $b = 15$ $c = 22$



$a^2 + 15^2 = 22^2$

$a^2 + 225 = 484$

$a^2 = 259$

$a = \sqrt{259}$

$a = 16.093$

$\sin B = \frac{15}{22}$

$B = \sin^{-1}\left(\frac{15}{22}\right)$

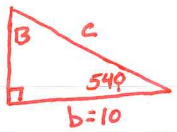
$B = 42.986^\circ$

$\cos A = \frac{15}{22}$

$A = \cos^{-1}\left(\frac{15}{22}\right)$

$A = 47.014^\circ$

b. $b = 10$ $A = 54^\circ$



$B = 180 - (90 + 54)$

$= 180 - 144$

$B = 36^\circ$

$\tan 54^\circ = \frac{a}{10}$

$10 \tan 54^\circ = a$

$13.764 = a$

$\cos 54^\circ = \frac{10}{c}$

$c \cos 54^\circ = 10$

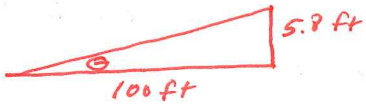
$\frac{c \cos 54^\circ}{\cos 54^\circ} = \frac{10}{\cos 54^\circ}$

$c = 17.013$

Avoid using Pythagorean Theorem for third side.

Ex: The grade of a road is 5.8%. What angle does the road make with the horizontal?

(vertical rise per 100 ft)



$\tan\theta = \frac{5.8}{100}$

$\theta = \tan^{-1}\left(\frac{5.8}{100}\right)$

$\theta = 3.319^\circ$

Ex: Solve the following equations:

a. $12 \tan\theta - 4 = 6$

$12 \tan\theta = 10$

$\tan\theta = \frac{10}{12}$

$\theta = \tan^{-1}\left(\frac{5}{6}\right)$

$\theta = 39.806^\circ$

b. $4 \sin\theta - 5 = 6 \sin\theta - 3$

$-5 = 6\sin\theta - 4\sin\theta - 3$

$-2 = 6\sin\theta - 4\sin\theta$

$-2 = 2\sin\theta$

$-1 = \sin\theta$

$\theta = \sin^{-1}(-1)$

$\theta = -90^\circ$ or 270°

HW: pg 299-300, #3-42 (m/3)